

UNITED STATES PATENT APPLICATION

of

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relating to a

SOFT BIT COMPUTATION FOR A REDUCED STATE EQUALIZER

SOFT BIT COMPUTATION FOR A REDUCED STATE EQUALIZER

Technical Field of the Invention

This invention relates generally to digital communication, and more particularly to low complexity equalization.

Background Art

For Time Domain Multiple Access (TDMA) wireless communications systems, an important problem is to mitigate intersymbol interference (ISI) caused when data is transmitted on a dispersive channel with accompanying background noise. This problem is different depending upon what type of air interface is employed. In a second generation system such as the global system for mobile communications (GSM) it suffices to treat the ISI problem in ways that are not adequate for third generation air interfaces such as EDGE (enhanced data rates for GSM evolution). In GSM, optimum performance in terms of error probability is typically accomplished by maximum likelihood sequence estimation (MLSE), but MLSE detection cannot be used to perform optimal equalization in EDGE, due to changes in both the modulation and the channel coding.

When discrete pulses or bursts of information are transmitted through a dispersive medium, they travel over multiple propagation paths to the receiver, and the information transmitted to the receiver will appear smeared. These multipath effects commonly result when signals are reflected to the receiver. The process implemented on the received signal to mitigate ISI is known as "equalization." This equalization process estimates the individual data symbols which were originally transmitted.

Various approaches have been taken to the equalization problem; linear equalization and decision-feedback equalization (DFE) are two common equalization strategies. However, DFE has poor performance in weak channel coding schemes,

such as EDGE transmission modes MCS8-9. A somewhat better approach has been based on MLSE, using the well-known Viterbi algorithm, although this approach has drawbacks as explained below. See *Shah* (U.S. Patent No. 6,134,277); *Cooper* (U.S. Patent No. 5,502,735).

5 Those skilled in the art will understand that a communication channel behaves in many respects like a filter, and therefore ISI itself may be modelled as a filter. The length of the filter is the extent of ISI, and this length may be referred to as the impulse response length L . Unlike DFE, MLSE detection typically uses trellis diagrams which show a progression of states with the passage of time, and the
10 number of states S is given by M to the power of $L-1$ (i.e. $S=M^{L-1}$) where M is the number of symbol levels in the data alphabet used. Consequently, for large response lengths and large alphabet sizes, the equalizer implementation becomes immensely complex. The complexity of the Viterbi Algorithm increases exponentially with channel length, and therefore implementation of the full Viterbi
15 Algorithm becomes unacceptably complex in EDGE. Thus, MLSE using the full Viterbi algorithm is inadequate, and a sub-optimal equalizer capable of performance comparable to that of MLSE is necessary, with acceptable low complexity.

 A well-known sub-optimal type of equalizer employs Reduced State Sequence Estimation (RSSE), wherein a reduced-state maximum likelihood (ML)
20 trellis uses set partitioning and decision feedback techniques. See, for example, "Reduced-State Sequence Estimation With Set Partitioning and Decision Feedback" by M. Eyuboglu, and S. Qureshi, IEEE Trans. Comm., vol. 36, pp. 12-20, January 1988. A particularly useful special case of RSSE is Delayed Decision Feedback Sequence Estimation (DFSE). A precondition for applying RSSE is that the input
25 signal is of minimum phase (i.e. the impulse response power decays in time), and this condition can be satisfied by applying prefiltering to the received signal, for example using a DFE feedforward filter.

 Additional equalization reliability measures are helpful in order to improve channel decoding subsequent to equalization, and for this purpose it is desirable to

generate soft-output information. Soft bits contain important reliability information that greatly aids the decoder located within the receiver downstream from the equalizer.

Soft-output information can be produced by an algorithm called symbol-by-symbol Maximum A Posteriori (MAP) decoding, which can calculate the probability of each symbol given the whole sequence that has been received. For example, see "Reduced State Soft-Output Trellis-Equalization Incorporating Soft Feedback," by Müller et al., Proc. IEEE GLOBECOM '96, November 1996. A max-log-MAP algorithm is familiar to persons skilled in the art as a simplified version of a MAP algorithm, and max-log-MAP equalizers operate in a logarithmic domain as compared to a probability domain. Another known method for producing soft-output information is using decision feedback partial sequence estimation (DFPSE). DFPSE equalizers are equalizers which employ a non-zero delay reduced state trellis calculation and which achieve reduction of implementation complexity by (1) employing decision feedback, (2) relying on improved accuracy of initial symbol detection which reduces the scope of searching, and (3) using ML or Max-log-MAP detection criteria. DFPSE is also known as block decision feedback equalization. See "Block Decision Feedback Equalization" by Williamson et al., IEEE Trans Comm., vol. 40, pp. 255-264, February 1992. As will be understood by those of ordinary skill in the art, non-zero delay refers to the those situations in which, after a signal is received, there must be a waiting period before soft bits are computed, and this may be due to the need to examine more than one received symbol.

Although using a reduced number of states "S" (e.g. via RSSE or DFPSE) produces very good hard symbol decisions with extremely low complexity, problems arise in producing soft decisions (i.e. soft bits) such as log likelihood ratios or bit probabilities of transmitted bits. A familiar response to these problems is to use trellis truncation, but trellis truncation in and of itself does not allow calculation of MAP or MLSE soft decisions for all bits of a symbol. In other words, when the number of states in the equalizer is reduced to obtain an acceptably

low complexity, computing soft bits for all of the bits becomes problematic according to existing methods.

The total number of states in a subset trellis is given by the product of the J_k and each J_k ranges from 1 to M . So, the minimum number of states for an equalizer would be 1 for $J=\{1,1,1...1\}$ which actually corresponds to DFE. At the other extreme is full MAP or MLSE in which case $J=\{J_1, J_2, J_3...J_{L-1}\}$ and $J_j = M$ and thus the number of states would be M^{L-1} as discussed above. Recall that "L" is impulse response length, and also bear in mind that the J_k are subject to a constraint $J_1 \geq J_2 \geq J_3 \geq ... J_{L-1}$. For an equalizer having $J=\{J_1, J_2, J_3...J_{K-1}, 1, 1...1\}$ where $K \leq L$ and $J_i < M$ the hard decision performance remains extremely good, but calculation of adequate soft decisions is difficult. Therefore, a need exists to increase reliable performance of low complexity equalizers, especially for high order modulations, by more fully and successfully producing soft bit information.

In general, EDGE systems involve capture of 2 or 3 taps in the MAP or MLSE part of the equalizer, and this means 8 or 64 states respectively for existing DFSE equalizers, or alternatively 4 or 16 states respectively for existing RSSE/RS-MAP equalizers with 4-way set partitioning. Digital signal processing (DSP) implementation of the simplest 4-state equalizer currently requires some 32 million instructions per second (MIPS) per time slot for EDGE at 8 PSK (phase shift keying). In order to provide higher data rate, it is desirable to have lower complexity algorithms than are currently available, or else DSP will be too costly. Although 2-way set partitioning with RSSE or RS-MAP is a candidate for achieving lower complexity, it entails problematic soft bit computation. Likewise, soft symbol equalization is a candidate for replacing soft bit equalization, but soft symbol equalization is approximately 1.5 dB worse than soft bit equalization for 8PSK EDGE, and this situation gets even worse for higher order modulations using current technology.

Disclosure of the Invention

The present invention provides a novel method of computing soft bits (which hereinafter refers to log likelihood ratios or bit probabilities) for any reduced state equalizer. This new method for making soft decisions uses low-complexity, while
5 providing soft bit information for substantially all received bits. This invention allows an equalizer to be implemented with a minimum number of states, while retaining high performance. This reduction of implementation complexity is accomplished by methods which, for example, combine MLSE or MAP soft decisions with either decision-feedback equalization (DFE) decision-making or with
10 other zero-delay soft decision-making.

The method involves computing a first set of soft bits for transmitted bits in a received signal, using a reduced-state trellis with finite non-zero delay, calculating hard decisions in response to the received signal, and also ensuring that substantially all soft bits are computed by employing zero-delay soft decision-making or decision-
15 feedback equalization to compute a second set of soft bits. Furthermore, the hard decisions are used to compute the second set.

This invention assures that the number of states in the equalizer is reduced to obtain an acceptable complexity, while also ensuring that soft bit computation is not omitted for a substantial number of bits. Therefore, this method increases the
20 reliability of low complexity equalization, especially for high order modulations, by more fully and successfully producing soft bit information.

According to the invention, soft bits can be computed from a received signal using a reduced-state trellis with MAP soft decisions, and hard decisions are also computed in response to the received signal. Zero-delay soft decision-making or
25 decision-feedback equalization is also employed, using the hard decisions to compute the soft values, and this ensures that soft bits are computed for substantially all bits. Decision-feedback equalization, using these hard decisions to compute the soft values, is unconventional and provides a marked improvement in soft bit computation.

The present method is flexible in that it can be used in a number of different equalization environments. For example, the trellis calculation may produce maximum a posteriori (MAP) soft decisions, and/or may produce soft decisions with reduced state sequence estimation (RSSE), reduced state MAP, and/or reduced state MLSE. The trellis calculation may utilize maximum likelihood (ML) techniques, and may produce soft decisions with decision feedback partial sequence equalization (DFPSE).

The present soft bit equalization invention not only offers a 1.5 dB gain (or better) as compared to soft symbol equalization, but also allows the number of states in the equalizer to be reduced to the minimum possible value. Note that different types of equalizers are not necessarily altogether different and distinct from each other, and there is often considerable commonality within subsets of equalizers. The present method is applicable not only to various types of equalizers, but also to arbitrary modulation sizes, and to any multiple access schemes such as TDMA/CDMA requiring a reduced state equalizer.

Preferable implementation modes for the present invention include digital signal processor (DSP) and/or the American standard code for information interchange (ASCII). According to current trends, wireless communication will employ higher and higher data density, and the present invention will probably be important for implementing trellis-based soft bit equalizers in any wireless transmission environment that is significantly dispersive, for example in a high density and high delay system like 16-64 QAM (quadrature amplitude modulation).

Brief Description of the Drawings

Figure 1 is a flow chart illustrating basic steps of the present method.

Figure 2 shows the structure and interactive cooperation within the present invention, in the context of a wireless communication receiver.

Figure 3 illustrates an embodiment of the present invention employing DFE and either RSSE or Max-Log-MAP.

Figure 4 depicts an aspect of an embodiment in which soft bits are computed symbol-by-symbol using with zero delay.

Figure 5 illustrates an embodiment of the present invention employing zero delay and either RSSE or Max-Log-MAP.

5 Figure 6 shows an embodiment of the present invention employing DFE and DFPSE.

Figure 7 shows a trellis for a two-state equalizer.

Figure 8 shows a trellis for a four-state equalizer.

10 Best Mode for Carrying Out the Invention

General features of a best embodiment of the present method are illustrated by the flow chart of Figure 1. The method involves computing **100** a first set of soft bits for bits transmitted in a received signal, using a reduced-state trellis with forward recursion and with finite non-zero delay. Alternatively, the computation
15 **100** may advantageously utilize a Viterbi Algorithm, such as a soft output Viterbi algorithm (SOVA). The method also involves calculating **110** hard decisions in response to the received signal, and also ensuring **120** that substantially all soft bits are computed by employing zero-delay soft decision-making or decision-feedback equalization to compute a second set of soft bits. The hard decisions are used to
20 compute the second set.

Although Figure 1 depicts a particular sequence for the steps, different sequences can be used in other best mode embodiments. The sequence shown in Figure 1 is desirable if we want to ensure that the first set of soft bits and the second set of soft bits are soft bits for two different sets of bits. In other words, once the
25 first set is computed **100**, then only soft bits for the remaining bits need to be computed **120** for the second set. The hard decisions are calculated **110** using the first set of soft bits. Using the hard decisions to compute soft bits is helpful to reduce error propagation.

In some applications, it will be useful to augment the method shown in Figure 1, by generating further hard decisions using a traceback operation, checking if the further hard decisions generated using the traceback operation match the hard decisions already calculated in step 110 using the first set of soft bits, and replacing soft bits generated using the trellis with the further hard decisions generated using the traceback operation, if the further hard decisions generated from the traceback operation fail to match respective hard decisions already calculated in step 110.

As shown in Figure 2, the present invention can be embodied in a wireless communications receiver 295 containing a reduced-state equalizer 200. The equalizer 200 is capable of providing soft bits to a decoder 290 in order to enhance reliability of channel decoding subsequent to equalization. The reduced-state equalizer includes a reduced-state trellis module 220, responsive to a received bit signal on a line 260, for providing a soft bit signal on a line 280 and a hard decision signal 270 on a line 270. The equalizer also includes a gap-filler module 210, responsive to the received bit signal and also responsive to the hard decision signal 270, for providing a second soft bit signal 292. The second soft bit signal 292 includes soft bits that are absent from the first soft bit signal, so that the first and the second soft bit signals include soft bits for substantially all bits in the received bit signal 260. In other words, the gap-filler module 210 produces soft bits which will fill any gaps in the soft bit computation performed by the reduced-state trellis module 220. The reduced-state trellis module 220 is configured to operate with forward recursion and finite non-zero delay, and the gap-filler module 210 is configured as a decision feedback equalizer or as a zero-delay soft decision-maker (i.e. configured for DFE or zero-delay soft decision-making). The decoder is responsive to the first soft bit signal 280 and the second soft bit signal 292, and is configured to provide a reliably decoded signal 297. Of course, inventive portions of the receiver and equalizer are shown in Figure 2 while other structures and signals are omitted from Figure 2 for the sake of clarity.

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In this embodiment, the finite non-zero delay employed by the reduced-state trellis module **220** is K minus one ($K-1$), where K is a truncated impulse response length and is less than or equal to a non-truncated impulse response length L (i.e. $K \leq L$). For the zero-delay soft decision-making, which may be employed by the gap-filler module **210**, K vanishes (i.e. $K=0$). Alternatively, if the gap-filler module **210** is configured for DFE, then the second set soft bits can be obtained with delay K minus two ($K-2$) if the trellis module **220** is configured for max-log-MAP. But, if the trellis module **220** is of an RSSE type then the gap-filler module **210** can have arbitrary delay. Only forward recursion is used in the best embodiment of the present invention, and thus forward recursion is sufficient without the typical backward recursion, although backward recursion is possible in other embodiments of the present invention.

Figure 3 is a flow chart illustrating a particular embodiment of the present invention, employing a DFE step **310** in order to fill gaps, and an RSSE/Max-Log-MAP step **320** to perform a reduced-state trellis calculation. Whereas Figure 1 showed a non-zero delay computation followed by a DFE computation, Figure 3 illustrates the DFE computation **310** occurring before the RSSE/Max-Log-MAP step **320** (which is a non-zero delay reduced state trellis calculation). Thus, in the best mode embodiment shown in Figure 3, a soft bit computed in the DFE step **310** for a certain bit is replaced with a soft bit computed for that bit in the RSSE/Max-Log-MAP step **320** if the soft bit for the bit is computed in the RSSE/Max-Log-MAP step **320**; otherwise the soft bit computed for the bit in the DFE step **310** will not be replaced and will effectively fill a gap in the results of the RSSE/Max-Log-MAP step **320**. Notice that Figure 3 shows a feedback step **360**, according to which hard symbol decisions are fed back from the RSSE/Max-Log-MAP step **320** to the DFE step **310**. The flow chart of Figure 3 also shows an MSSE-DFE prefiltering step **300** which precedes the other steps just discussed (MMSE stands for minimum mean-squared error), and a channel decoding step **340** which follows the other steps just discussed. Between the decoding step **340** and those other steps discussed, it is

useful to include a soft bit output step **330** in order to perform the previously described replacement of soft bits calculated in the DFE step **310** soft bits calculated in the RSSE/Max-Log-MAP step **320** (a soft bit output module could be included upstream from the decoder or within the decoder). Regardless of when, or in what order, the non-zero delay reduced state trellis calculation is performed, it is useful in a best mode embodiment to perform this calculation for as many bits as possible while maintaining acceptable implementation complexity.

In Figure 3, regarding the feedback step **360** in which hard symbol decisions are fed back from the RSSE/Max-Log-MAP step **320** to the DFE step **310**, the hard decisions may be computed using the max-log maximum a posteriori technique. Subsequently, the decision feedback equalization step **310** will be advantageously performed according to a rule described mathematically as follows:

$$\beta_{DFE}(a_k) \approx \left\| r_k - (a_k h_o + \sum_{j=1}^{L-1} b_{k-j} h_j) \right\|^2 \quad (1)$$

in which beta (β) represents a log symbol probability function, a_k represents a symbol of a modulation alphabet, r_k is a received signal, b_k are the hard decisions, and h_k is channel impulse response length. A decision produced by the DFE step **310** can be used to limit the number of states involved in the RSSE/Max-Log-MAP step **320**, but that is not necessary in a best mode embodiment.

Figure 4 depicts an aspect of the invention in which soft bits are computed symbol-by-symbol using zero delay. First a received signal is subjected to an MMSE-DFE prefilter step **400**, then is processed in a RSSE symbol-by-symbol zero-delay soft decision step **410**, and finally is decoded in a channel decoder step **420**.

The RSSE Symbol-by-symbol zero-delay soft decision step **410** advantageously employs a trellis calculation, and implemented with a maximum complexity that is twice as complex as implementation of decision-feedback equalization would be. This RSSE Symbol-by-symbol zero-delay soft decision step

410 will preferably be performed according to a rule described mathematically as follows:

$$\beta_{0delay}(s_{k+1} = a_k) \approx \min_{(s_k)} \{\mu(s_k) + \lambda(s_k, s_{k+1})\} \quad (2)$$

- 5 in which beta represents a zero-delay log symbol probability function, s_k denotes a previous symbol decision, s_{k+1} denotes a current symbol decision, a_k represents a symbol of a modulation alphabet, min is a minimizing function, mu (μ) and lambda (λ) represent two paths connecting two states.

Figure 5 illustrates an embodiment of the present invention employing zero delay and either RSSE or Max-Log-MAP. This Figure 5 is similar to the embodiment shown in Figure 3, the primary difference being that the DFE step 310 of Figure 3 is replaced by the symbol-by-symbol zero-delay soft decision step 410 of Figure 4. The flow chart of Figure 5 begins with a prefiltering step 500, followed by a symbol-by-symbol zero-delay soft decision step 510 and an RSSE/Max-Log-MAP soft decision step 520 (steps 510 and 520 can occur in either order), and then a soft bit output step 530 followed by a decoding step 540.

When the non-zero delay trellis computation involves max-log-MAP, as may occur in step 320 of Figure 3 or step 520 of Figure 5, further efficiency is obtained if first and second subsets of signals are established by Euclidean partitioning so as to reduce trellis size to two states corresponding to the first and second subsets respectively. Then the more likely of the two states is determined, and Euclidean distances are computed for all paths corresponding to the more likely state; only the paths corresponding to the more likely state would then be used to perform trellis computation of soft bits in step 510 of Figure 5. A decision produced by the symbol-by-symbol zero-delay soft decision step 510 can be used to limit the number of states involved in the RSSE/Max-Log-MAP step 520, and that is necessary in a best mode embodiment of the present invention.

Figure 6 shows another best mode embodiment of the present invention, employing DFE and decision feedback partial sequence equalization (DFPSE).

Figure 6 is, again, similar to the embodiment shown in Figure 3, the primary difference being that the RSSE/Max-Log-MAP step **320** of Figure 3 is replaced by a DFPSE step **620** which (again) involves a non-zero delay reduced state trellis calculation. The flow chart of Figure 6 begins with a prefiltering step **600**, followed by a DFE step **610** and a DFPSE step **620**. Steps **610** and **620** can occur in either order, and in either case a feedback step **660** feeds hard symbol decisions from DFPSE to DFE. Decisions produced by DFE can be used to reduce the number of states in the DFPSE trellis. The final steps in this embodiment are soft bit output **630** followed by decoding **640**.

It should be emphasized that in all of the best mode embodiments shown in the figures, a non-zero delay reduced state trellis step computes soft bits and hard symbol decisions, either before or after a DFE/zero-delay step also computes soft bits. Furthermore, this DFE/zero-delay step always utilizes the hard symbol decisions to compute soft bits. The present invention thus ensures that, in one way or another, soft bits are efficiently calculated for substantially all bits transmitted in a received signal.

Those skilled in the art will understand that each of the best mode embodiments described herein can be implemented in a variety of ways, by a variety of different combinations of hardware and software, and will further understand that the structures, steps, and interactions described herein are conceptual elements that likewise can be implemented by various combinations of hardware and software and integrated with a variety of other structures, steps, and interactions.

Generally speaking, the present invention includes novel methods of soft bit computation for any reduced state equalizer. Although perhaps somewhat repetitive, it may be useful to now discuss further some general characteristics and examples of the invention in the context of EDGE at 8PSK. We shall consider soft bit generation algorithms for a reduced state set partitioning Max-Log-MAP equalizer using 8PSK as an example, although the same ideas are also applicable to

RSSE or DFPSE equalization. As an example we will use a 2-state Max-Log-MAP equalizer for EDGE 8-PSK in which $J=\{2,1,1,1,1\}$.

Let us consider 2—state, 2-way set partitioning Max-Log-MAP soft bit detection. This example will have three phases, but first some preliminaries.

- 5 Divide the 8PSK signal constellation $\{0,1..7\}$ into two subsets A and B where $A=\{0,2,4,6\}$, and $B=\{1,3,5,7\}$. The subsets are chosen such that the minimum Euclidean distance between any points in the subset is maximized, as discussed in the article by Eyuboglu and Qureshi. This type of partitioning is called Euclidean partitioning. The subset trellis for a 2-tap equalizer ($K=2$) is shown in Figure 7. A
- 10 normal Viterbi trellis contains 8 states corresponding to 8 signal points. In the present case, the trellis size is reduced to two states corresponding to two sub sets A and B. Note from Figure 7 that there are parallel transitions merging at each node. Since the Euclidean distance between the points in a given subset is maximized, early merging paths can be reliably distinguished. A minimum phase condition is
- 15 required when making such early decisions. Now we would like to incorporate forward Max-Log-MAP detection through this reduced state trellis. The prefiltered-received signal sampled at symbol rate can be written as:

$$r(k) = a_0 h_0 + \sum_{j=1}^{L-1} a_{k-j} h_j + n(k) \quad (3)$$

- Where a_k is the 8PSK modulation alphabet, h_k is the channel impulse response of
- 20 length L , and $n(k)$ is the thermal noise. To reduce complexity, the first K taps are used in the main part of the trellis, and decision feedback is used for the remaining taps. Then the number of states in the trellis is given by 2^{K-1} . In Figure 7, the number of states is restricted to 2. Referring to Figure 7, a symbol a_k arrives at $t=kT$, and s_k denotes the state at $t=kT$. The state labeling is given by $s_k = \hat{a}_{k-1}$ and
 - 25 s_k contains the previous symbol decision. Furthermore, $s_{k+1} = \hat{a}_k$ and s_{k+1} denotes the current symbol decision.

The first phase in this 2-state, 2-way set partitioning Max-Log-MAP soft bit detection example begins by adding the path metric $\lambda(s_k, s_{k+1})$ to the old path

metric $\mu(s_k)$ for all the paths merging at a given state s_{k+1} , and choosing the minimum distance winner path. Then store \hat{a}_k for this winner path and update the path history. This is denoted by the following equation:

$$\mu(s_{k+1}) \approx \min_{(s_k)} \{\mu(s_k) + \lambda(s_k, s_{k+1})\} \quad (4)$$

where s_k denotes all the states corresponding to previous symbol \hat{a}_{k-1} . In Figure 7, there are 8 branches leaving each node $s_k = \hat{a}_{k-1}$, and 8 branches merging at each node $s_{k+1} = \hat{a}_k$. The state dependent path history contains all previous L-K decisions on \tilde{a}_k . The path metric for each branch connecting (s_k, s_{k+1}) is defined as

$$\lambda(s_k, s_{k+1}) = \frac{\left\| r_k - (a_k h_0 + \hat{a}_{k-1} h_1 + \sum_{j=2}^L \tilde{a}_{k-j} h_j) \right\|^2}{2\sigma^2} \quad (5)$$

In equation (5), \hat{a}_k is obtained from the previous state s_k , and \tilde{a}_k is the path history stored in s_k .

The second phase in this example is max-log-MAP symbol detection with 1-symbol delay. In Figure 7, decisions on symbol a_k are made with 1-symbol delay at $t=(k+1)T$ meaning at state s_{k+1} . According to forward MAP criterion (forward MAP is used in this example but the invention is also applicable when backward recursion is included), it is known in the art that the state probabilities are given by:

$$P(s_{k+1}) = \sum_{s_{k+2}} \exp[-\{\mu(s_{k+1}) + \lambda(s_{k+1}, s_{k+2})\}] \quad (6)$$

in which s_{k+2} denotes all the paths leaving state s_{k+1} . In the example of Figure 7, there are eight branches leaving each state. The summation in equation (6) is performed for all 8 branches leaving s_{k+1} . To make a decision on symbol \hat{a}_k we need to decide whether \hat{a}_k belongs to subset A or subset B. First compute,

$$P(s_{k+1} = A) = \sum_{s_{k+2}} \exp[-\{\mu(s_{k+1} = A) + \lambda(s_{k+1} = A, s_{k+2})\}] \quad (7)$$

$$P(s_{k+1} = B) = \sum_{s_{k+2}} \exp[-\{\mu(s_{k+1} = B) + \lambda(s_{k+1} = B, s_{k+2})\}] \quad (8)$$

The summation is performed for $s_{k+2} = A$ and $s_{k+2} = B$. The hard decision rule to detect symbol \hat{a}_k is given by this rule: if $P(s_{k+1} = A) > P(s_{k+1} = B)$, then \hat{a}_k

- 5 belongs to state A. This means that the current symbol stored in state A will be the detected symbol. In practice it is easier to deal with log symbol probabilities for complexity reduction purposes. Define beta as follows:

$$\beta(s_{k+1} = A) = -\ln[P(s_{k+1} = A)]. \quad (9)$$

- 10 The presence of exponentials in equation (6) is computationally expensive, and we resort to the Max-Log approximation for numerical simplification. The negative logarithm of a sum of exponentials can be approximated as:

$$-\ln(\sum_j e^{-\delta_j}) = \min(\delta_1, \delta_2, \dots, \delta_j) \quad (10)$$

Then equation (9) can be simplified as:

$$15 \quad \beta(s_{k+1} = A) \approx \min_{(s_{k+2})} \{\mu(s_{k+1} = A) + \lambda(s_{k+1} = A, s_{k+2})\} \quad (11)$$

The third phase in this example is soft bit generation. In order to take full advantage of soft bit decoding gain, we need to convert the detected symbol bit probabilities to bit probabilities. Let us assume that state s_{k+1} takes two possibilities A=0 and B=1. Symbols 0 and 1 are mapped to the bits (111) and (011) respectively. Then the bit probabilities can be written as:

$$Pb_0(0) \approx 0 \quad \text{and} \quad Pb_0(1) = P(0) + P(1) \approx 1.0 \quad \text{and} \quad Pb_1(0) \approx 0 \quad (12)$$

and

$$Pb_1(1) = P(0) + P(1) \approx 1.0 \quad \text{and} \quad Pb_2(0) = P(1) \quad \text{and} \quad Pb_2(1) = P(0) \quad (13)$$

where $Pb_i(0)$ denotes the probability of the i 'th bit position being zero. It is clear that we can generate appropriate bit probabilities for the most significant bit (MSB), and the remaining two bit positions can have hard decisions only. Since it is computationally efficient to deal with log bit probabilities, we use the following
5 definition for bit log-likelihood-ratios (LLR):

$$q_i = -\ln(Pb_i(1)/Pb_i(0)) \quad (14)$$

Then the LLR for each bit position can be written as:

$$10 \quad q_0 \approx \pm\infty \quad \text{and} \quad q_1 \approx \pm\infty \quad \text{and} \quad q_2 \approx \beta(1) - \beta(0) \quad (15)$$

In short, 2-state, 2-way set partitioning Max-Log-MAP soft bit detection can be described in the following phases. First, compute Euclidean distances for all the 16-paths originating from nodes A and B. Second, for the MSB, search for a
15 minimum distance among all 16 paths that have a zero in the MSB position, and repeat this process for bit 1 in the MSB position (soft information for the MSB is defined as the difference between these two minimum distances). Third, repeat the second phase for all other bit positions.

For computational efficiency, we generate soft bit information directly from
20 the Euclidean distances. We avoid the intermediate step of symbol probability generation. In this particular example, we can generate soft bit information for the MSB only; the other two bits remain in an initialization state causing soft decision failure. The present invention conveniently includes several solutions that will remedy this 2-way set partitioning Max-Log-MAP soft bit detection failure
25 problem. Further simplification can be obtained by calculating one Euclidean distance for all the parallel transitions between two nodes: first make a hard decision using a slicing operation, and then compute one Euclidean distance for the part corresponding to the hard decision. This reduces implementation complexity in both the Max-Log-MAP and the Viterbi part of the algorithm.

A first "soft decision feedback" solution to this 2-way set partitioning max-log-MAP soft bit detection failure problem can be described as follows. Generate hard decisions b_k from RSSE/RS-Max-Log-MAP. Then run DFE with RSSE/RS-Max-Log-MAP hard decisions in the feedback part. For bits that remain in an initialization state, generate soft bit information for those remaining bits. For the MSB in the given example, use Max-Log-MAP. This first solution can best be understood by again referring to Figure 7. At state s_{k+1} use RSSE/Max-Log-MAP to create soft bit information on symbol a_k for possible bits, and to create soft information for the remaining bits run DFE on the received signal r_k with Max-log-MAP hard decisions b_k as feedback. This leads us back to equation (1) above, and this "soft decision feedback solution" generally corresponds to the flow chart in Figure 3.

A second solution to this 2-way set partitioning max-log-MAP soft bit detection failure problem may be called the "symbol-by-symbol soft bit" solution, and it generally corresponds to the flow chart in Figure 4. In this solution, symbol-by-symbol soft bit information is generated in the trellis without any delay. Referring again to Figure 7, search for soft bit information for all the 16 paths arriving at s_{k+1} . Use the rule given by equation (2) above. In equation (2), to detect symbol $a_k = 0$, minimization is performed for the sixteen paths originating from s_k . The complexity of this method is exactly twice that of DFE.

A third solution to this 2-way set partitioning max-log-MAP soft bit detection failure problem may be called the "improved soft bit" solution, and it generally corresponds to the flow chart in Figure 5. In this solution, we utilize the improvements from both the first "soft decision feedback" solution, and from the second "symbol-by-symbol soft bit" solution. Begin by generating possible soft bit information using Max-Log-MAP. Then use the second "symbol-by-symbol soft bit" solution to generate soft information for the remaining bits. Referring again to Figure 7, at state s_k store the Euclidean distance for all the 16 paths. During the

next symbol interval (i.e. at state s_{k+1}), generate soft bit information for the MSB.

Then generate soft information for the remaining bits by searching through the 16 paths stored in the previous symbol interval. To further reduce the complexity of this "improved soft bit" solution, Max-Log-MAP detects the symbol a_{k-1} at state

5 s_k . Then, store the 8-paths originating from $s_k = a_{k-1}$ only, instead of all 16-paths. We only need to search through these 8-paths for computing the soft bit information. Instead of using max-log-MAP, one can use an RSSE/MLSE soft output Viterbi algorithm (SOVA) to compute soft bit information. However, if soft bit computation is not possible for a particular bit, then we can replace it using the
10 "soft decision feedback solution" or using zero-delay.

A fourth solution to this 2-way set partitioning max-log-MAP soft bit detection failure problem may be called the "max-log-MAP with L symbol delay" solution. So far, we have implemented max-log-MAP with K symbol delay (i.e. one symbol delay for the 2-state case shown in Figure 7). We can get some extra
15 gain by delaying max-log-MAP decisions by an amount greater than the K symbol delay.

A fifth solution to this 2-way set partitioning max-log-MAP soft bit detection failure problem may be called the "solutions one to four plus traceback" solution. It is generally believed that the hard decisions given by the traceback operation contain
20 fewer errors than solutions 1-4. Hence, we have proposed, in the present invention, to generate hard decisions from traceback, and check if these decisions match the hard decisions from solutions 1-4. If not, replace the soft bit information from solutions 1-4 with the traceback hard decisions.

Up until now, the example we have been considering has involved a 2-state
25 Max-Log-MAP equalizer for EDGE 8-PSK in which $J=\{2,1,1,1,1\}$, and we have been considering 2-way set partitioning Max-Log-MAP soft bit detection. But suppose we have a 4-state Max-Log-MAP equalizer, and let us consider 2-way set partitioning and soft bit detection by analogy with the two-state example. Instead of Figure 7, we must now refer to the 3-tap, 4-state trellis diagram of Figure 8. In

Figure 8, each state s_k can have four possibilities AA, AB, BA, BB where A takes one of the four symbols from the subset $\{0,2,4,6\}$, and B takes values from the subset $\{1,3,5,7\}$. The state labeling is given by:

$$s_k = (\hat{a}_{k-2}, \hat{a}_{k-1}) \quad \text{and} \quad s_{k+1} = (\hat{a}_{k-1}, \hat{a}_k) \quad (16)$$

There are two phases in this 4-state, 2-way set partitioning Max-Log-MAP soft bit detection example. The first step utilizes an equation which is a slight revision of equation (4):

$$\mu(s_{k+1}) \approx \min_{(s'_k)} \{ \mu(s'_k) + \lambda(s'_k, s_{k+1}) \} \quad (17)$$

Equation (17) denotes that for all the paths merging at a given state s_{k+1} , add the path metric $\lambda(s'_k, s_{k+1})$ to the old path metric $\mu(s'_k)$, and choose the minimum distance winner path. We introduce the notation s'_k to denote all the paths originating from nodes s'_k and merging at node s_{k+1} . The path metric is defined by an equation which is a slight revision of equation (5):

$$\lambda(s'_k, s_{k+1}) = \frac{\left\| r_k - (a_k h_0 + \sum_{j=1}^{K-1} \hat{a}_{k-j} h_j + \sum_{j=K}^{L-1} \tilde{a}_{k-j} h_j) \right\|^2}{2\sigma^2} \quad (18)$$

where \hat{a}_k is obtained from the previous state s'_k , and \tilde{a}_k is the path history stored in s'_k . From Figure 8, all the paths merging at $s_{k+1} = AA$ can have transitions originating from $s'_k = AA$ and $s'_k = BA$ only. In this case equation (17) can be written as:

$$\mu(s_{k+1} = AA) \approx \min_{(s'_k)} \{ \mu(s'_k) + \lambda(s'_k, AA) \} \quad (19)$$

where s'_k takes two possibilities (AA and BA). After determining the winner path, update the state-dependent path history vector \tilde{a}_k .

The second phase of this 4-state, 2-way set partitioning Max-Log-MAP soft bit detection example requires that the state probabilities are given by:

$$P(s_{k+K-1}) = \sum_{s'_{k+K}} \exp[-\{\mu(s_{k+K-1}) + \lambda(s_{k+K-1}, s'_{k+K})\}] \quad (20)$$

- 5 according to forward MAP criteria. For $K=3$, as is evident from Figure 8, equation (20) can be written as:

$$P(s_{k+2}) = \sum_{s'_{k+3}} \exp[-\{\mu(s_{k+2}) + \lambda(s_{k+2}, s'_{k+3})\}] \quad (21)$$

- 10 In equation (21), s'_{k+3} denotes all the paths leaving nodes s_{k+2} and terminating at s'_{k+3} . For example, if $s_{k+2} = AA$, then s'_{k+3} takes two possibilities: AA and BA. There are a total of eight branches leaving $s_{k+2} = AA$ and terminating at $s'_{k+3} = AA$ or $s'_{k+3} = BA$. Accordingly:

$$P(s_{k+2} = AA) = \sum_{s'_{k+3}} \exp[-\{\mu(s_{k+2} = AA) + \lambda(s_{k+2} = AA, s'_{k+3})\}] \quad (22)$$

$$P(s_{k+2} = BA) = \sum_{s'_{k+3}} \exp[-\{\mu(s_{k+2} = BA) + \lambda(s_{k+2} = BA, s'_{k+3})\}] \quad (23)$$

- 15 Then we find that:

$$P(s_{k+2} = A) = P(s_{k+2} = AA) + P(s_{k+2} = BA) \quad (24)$$

Similarly:

$$P(s_{k+2} = B) = P(s_{k+2} = AB) + P(s_{k+2} = BB) \quad (25)$$

- 20 The final decision on symbol \hat{a}_k would be one of the two states A or B, whichever is more probable. Since we are interested in the soft bit information, we need to compute bit level soft information. This is done exactly same way as described above in the two-state example. Moreover, equations (21) - (24) can be generalized to any set partitioning type, and to an arbitrary number of taps, in a straightforward way.
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Although this invention has been shown and described with respect to best mode embodiments and specific examples thereof, it should be understood by those skilled in the art that the foregoing and numerous other changes, omissions and additions in form and detail may be made therein without departing from the spirit
5 and scope of the present invention.

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